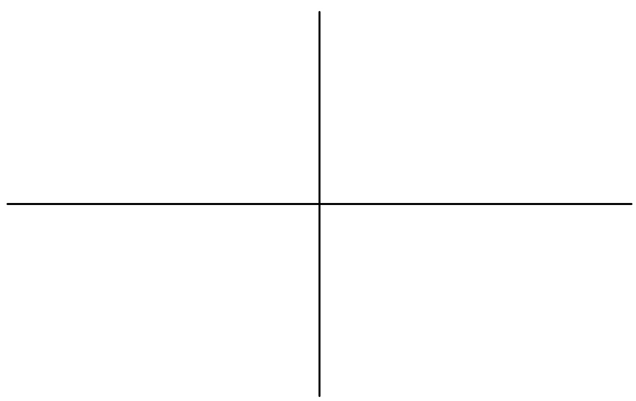
Polynomial functions: End behavior NAME:

In this lab, we are looking at the end behavior of polynomial graphs, i.e. what is happening to the *y* values at the (left and right) ends of the graph.

In other words, we are interested in what is happening to the *y* values as we get really large *x* values and as we get really small *x* values.

Recall a polynomial function is one that can be written in the form
$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ where *n* is the degree of the polynomial (must be a nonnegative integer), $a_n, \ldots, a_0$ are the coefficients, and a_n is called the leading coefficient.

To get an understanding of how we will denote end behavior, let's look at $y = x^3$. Quickly sketch a graph of it. (It would be nice to be able to do from memory, but use your grapher if you need.)

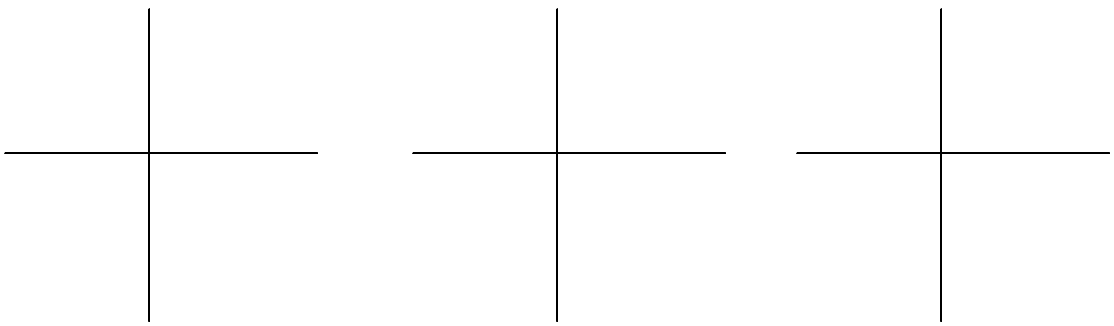


Notice how the *y* values soar off toward infinity on the right end (as *x* values get really big) and the *y* values soar off toward negative infinity on the left end (as *x* values get really small).

We denote this by writing "as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$". This is read "as *x* approaches negative infinity, *y* approaches negative infinity and as *x* approaches (positive) infinity, *y* approaches (positive) infinity". Notice the first part of this talks about the left end of the graph and the second part of this talks about the right end of the graph.

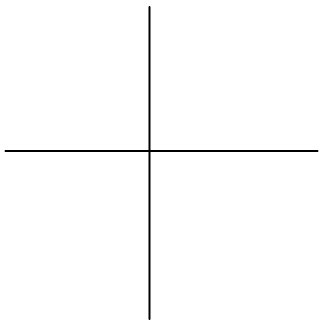
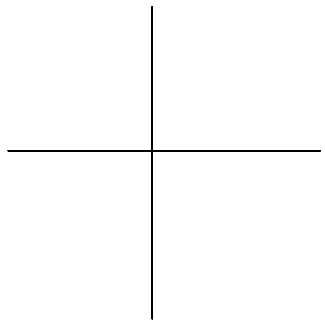
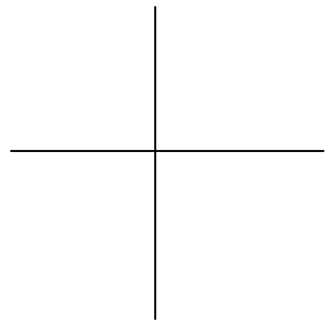
This worksheet will guide you through looking at the end behaviors of several polynomial functions. At the end, we will generalize about all polynomial functions. A good window for all the graphs will be [-10, 10] x [-25, 25] unless stated otherwise. Try to mimic the general shape and the end behavior of the graphs. However, you do not need to be very accurate as to where the x and y intercepts are.

1. Graph $y = 2x^3 + 4x$, $y = 3x^5 - 6$, and $y = x^7 + 2x^3 - 5x^2 + 2$ on the three planes below.



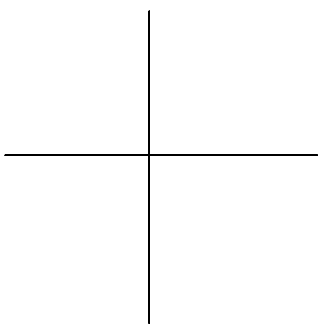
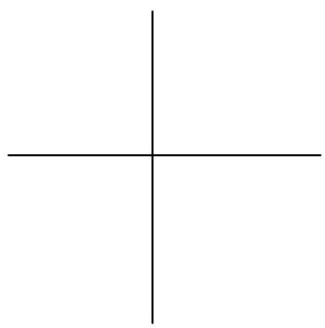
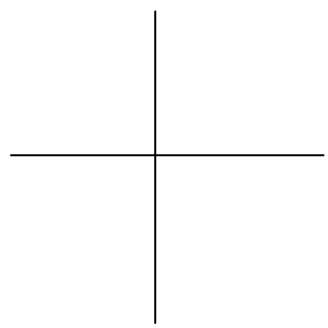
What is the end behavior of all three graphs above? Use the notation demonstrated on the first page.

2. Graph $y = -3x^5 - 5x^2 + 3x - 4$, $y = -4x^3 + 2x^2 + 3$, and $y = -x^7 + 2x^2 + 3x - 4$ on the three planes below.

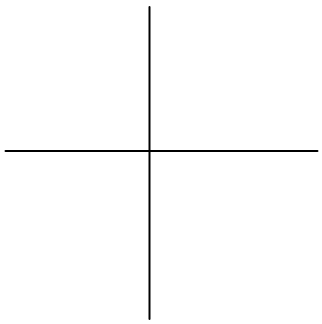
What is the end behavior of all three graphs above? Use the notation demonstrated on the first page.

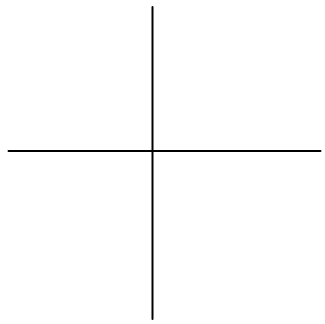
3. Graph $y = 2x^4 - 6x - 4$, $y = x^4 - 11x^3 + 42x^2 - 64x + 32$, and $y = x^6 + 5x^5 - x^4 - 21x^3$ on the three planes below. The window for the third one should be set to [-10, 10] x [-50, 75].

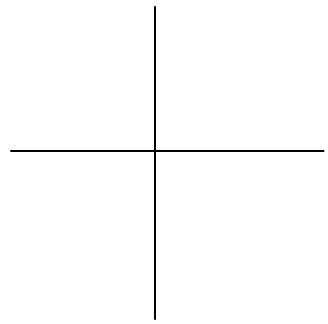




What is the end behavior of all three graphs above? Use the notation demonstrated on the first page.

4. Graph $y = -2x^4 + 3x^2 - 5$, $y = -2x^6 + 5x^3 - 4x + 8$, and $y = -x^4 + 3x^3 - 4x^2 + 7$ on the three planes below.





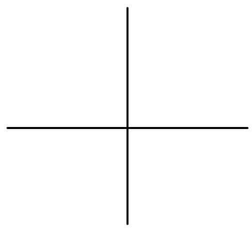


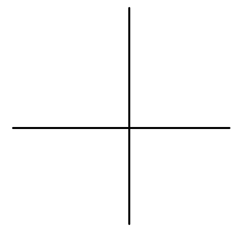
What is the end behavior of all three graphs above? Use the notation demonstrated on the first page.

5. Questions one through four gave you three examples of each kind of (polynomial) end behavior. **The two things that determine the end behavior of a polynomial are the degree (whether it's even or odd) and the leading coefficient (whether it's positive or negative). Look over your work for questions one through four to verify this.** Use the table below to summarize the end behaviors of polynomials. Use the notation demonstrated on the first page.

	Leading coefficient is negative	**Leading coefficient is positive**
Degree is odd		
Degree is even		

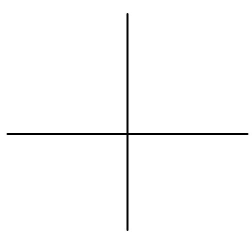
6. Way to remember end behavior of polynomials: I remember end behaviors by keeping pictures of $y = x^2$ and $y = x^3$ in my head. Draw these two functions now. (You should be able to do so from memory.) What are the end behaviors of these functions? Use the notation demonstrated on the first page.





I remember that all polynomials of even degree and positive leading coefficient have the same end behavior as $y = x^2$. And all polynomials of odd degree and positive leading coefficient have the same end behavior as $y = x^3$.

For polynomials of even degree and negative leading coefficient, I picture $y = -x^2$ which is a reflection of $y = x^2$ about the *x*-axis. Draw $y = -x^2$ now, using this information. What is the end behavior of $y = -x^2$? **All polynomials of even degree and negative leading coefficient share this same end behavior.**



For polynomials of odd degree and negative leading coefficient, I picture $y = -x^3$, which is a reflection of $y = x^3$ about the *x*-axis. Draw $y = -x^3$ now, using this information. What is the end behavior of $y = -x^3$? **All polynomials of odd degree and negative leading coefficient share this same end behavior.**